Session 3: An application to coding theory and cryptography

Oriol Serra

CIMPA-Indonesia School

Overview

Cycle codes of graphs

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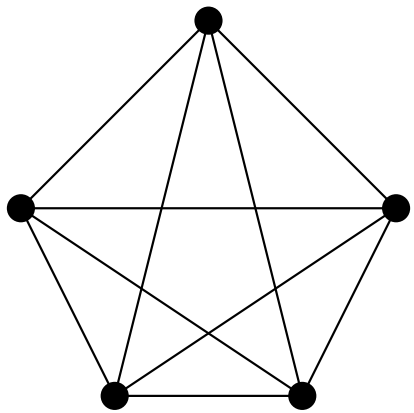
- $G = (V, E)$ undirected connected graph with no loops, no multiple edges.



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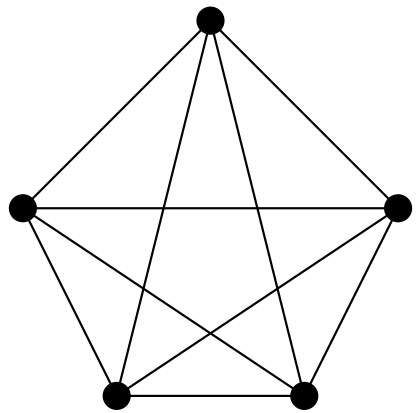

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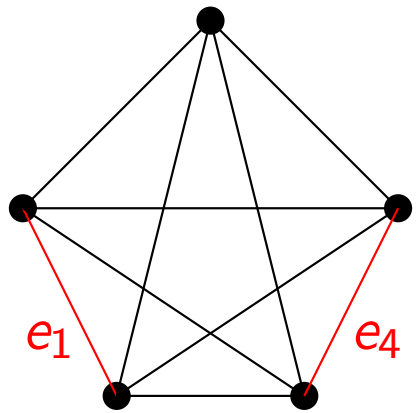

$$H = (1, 0, 0, 1, 0, 0, 0, 0, 0, 0)$$

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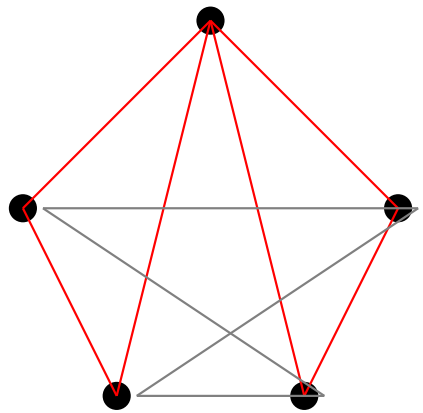

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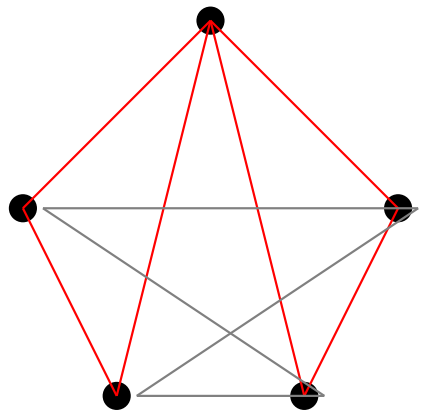
- Cycle $c \in \{0,1\}^m$: Subgraph with all vertices incident with an even number of edges.
- Cycle space of G: the vector space over $\mathbb{F}_2$ of all cycles. It has dimension $m - n + 1$ (cyclomotic number)



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Cycle codes of graphs

Cycle code of G: the linear binary code $[n, k, d]$ defined by the cycle space of G with

(i) length m (number of edges)

(ii) dimension $k = m - n + 1$ (cyclotomic number)

(iii) minimum distance $d =$ girth of G (length of smallest cycle).

(iv) incidence matrix of $G \longleftrightarrow$ parity-check matrix of the code (low–density parity-check code)

Array codes

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Binary Code of length $N = ab$ $\longrightarrow$ Array $a \times b$

- (100010001) $\longrightarrow$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

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- Array codes are used to address bursts of errors (as opposite to random errors).
  The are implemented in the standards of CD technology (by using Reed–Solomon codes).

L.B. Vries and K. Odaka, CIRC–The error correcting code for the Compact Disk digital audio system, Digital Audio, B.A. Blesser et al. editors, Audio Engrg. Soc. (1083), 178–186

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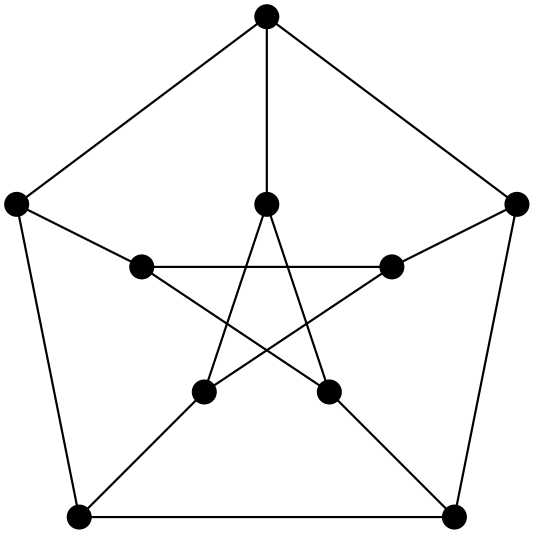
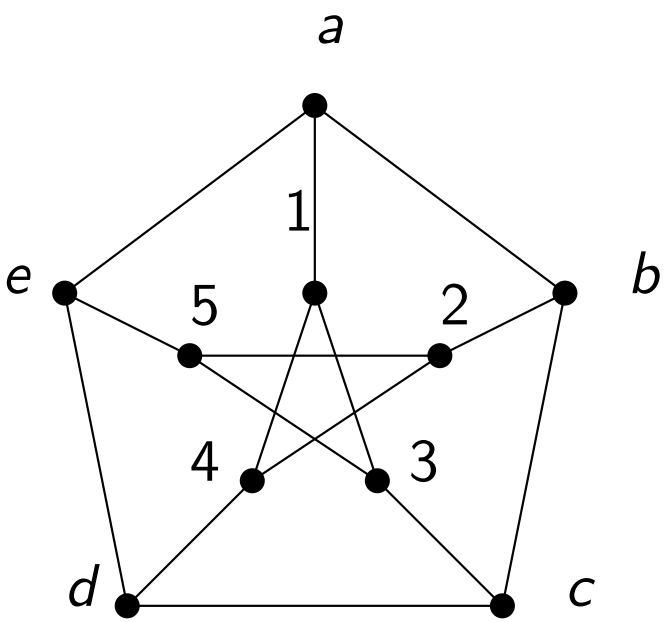


Array cycle codes

- Graph $G = (V, E)$ with $m = ab$ edges.
- Partition the edges in columns

$$\begin{pmatrix} a1 & b2 & c3 & d4 & e5 \\ bc & cd & de & ea & ab \\ 52 & 13 & 24 & 35 & 41 \end{pmatrix}$$



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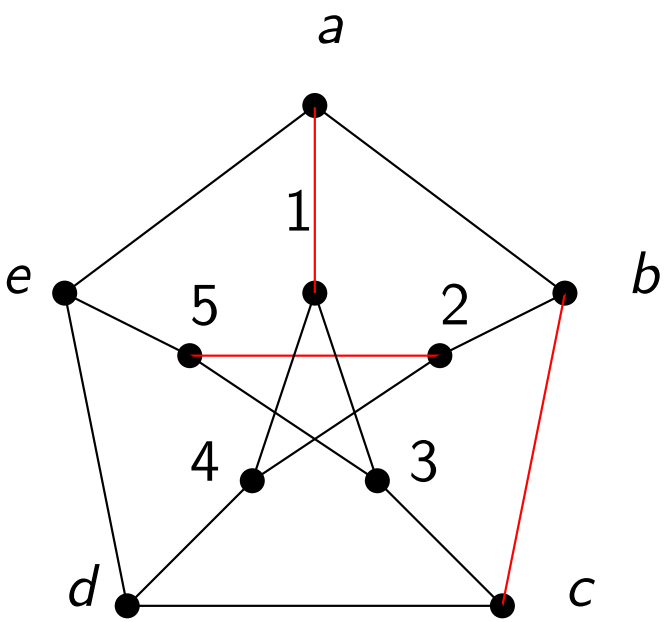

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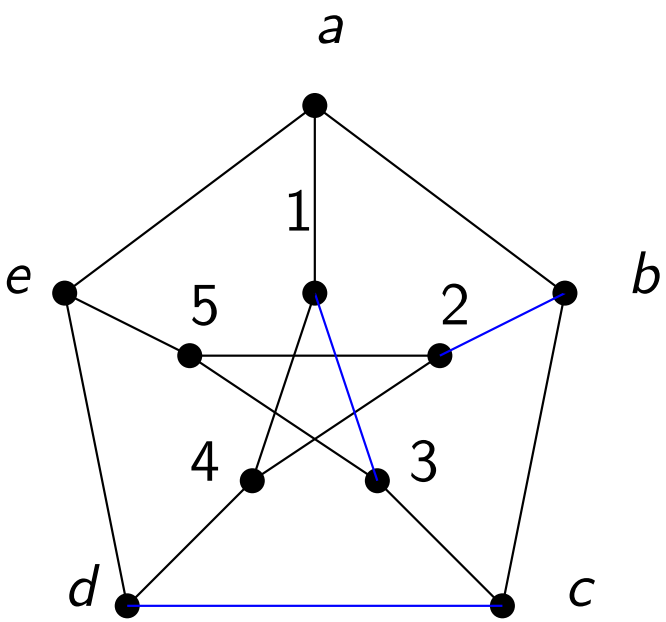

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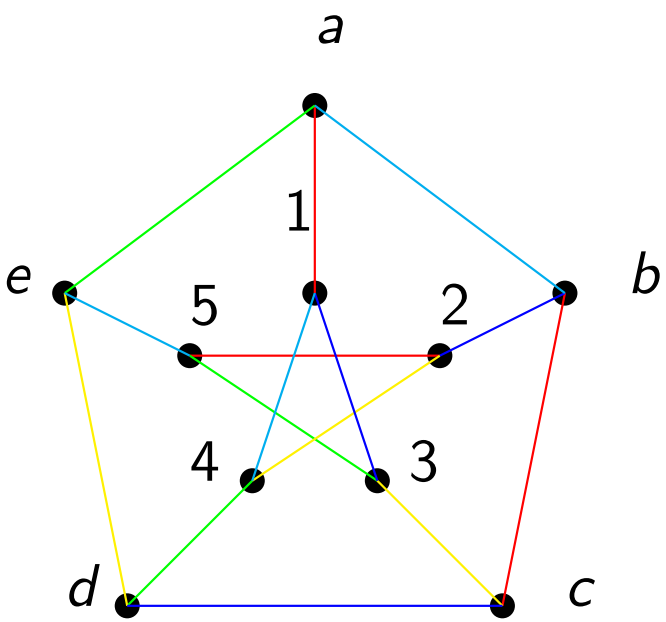

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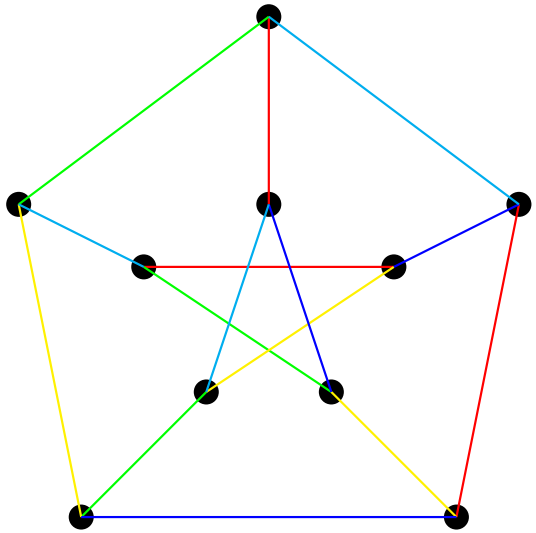
- Array Cycle code: The graph cycle code turned into an array code.



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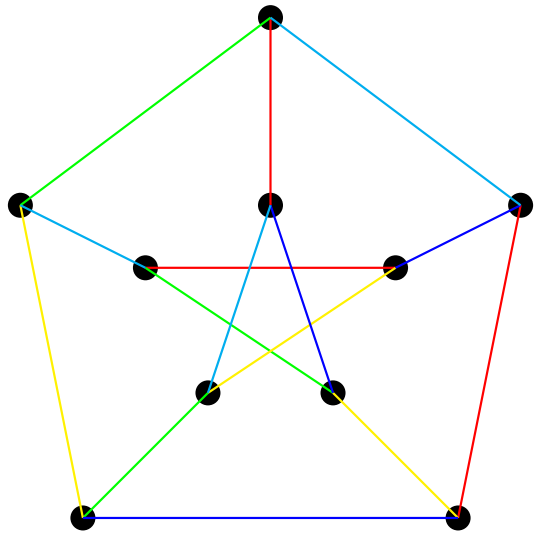

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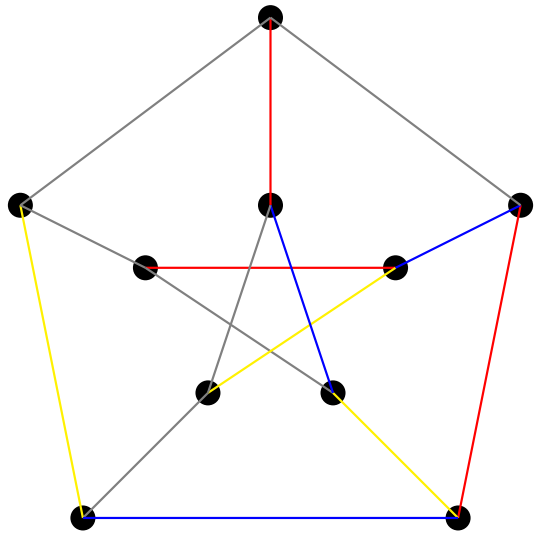
$$D \leq b - \log_{2^a} |C| + 1.$$

In the example, the code has $|C| = 2^{15-10+1}$ and $D = 4 = 5 - \log_8 |C| + 1$.
It is an MDS (Maximum Distance Separating) code.

This means that every three colors span a spanning tree.



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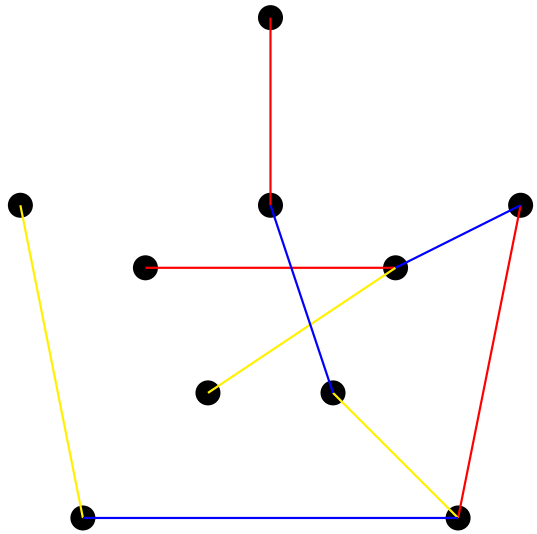

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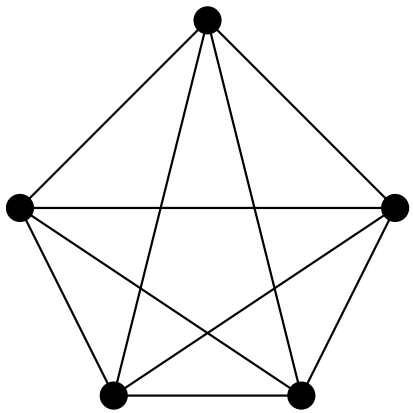

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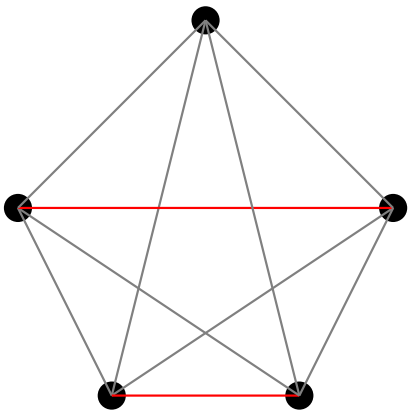

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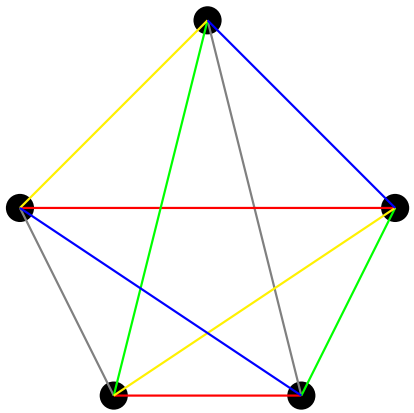
- They provide MDS array cycle codes with $a = (n-1)/2$, $b = n$, $\mathbb{F}_2$–dimension $n - 2a$ and $D = 3$.
  Known in the literature as B–codes.



Correction algorithms: Column erasure correction

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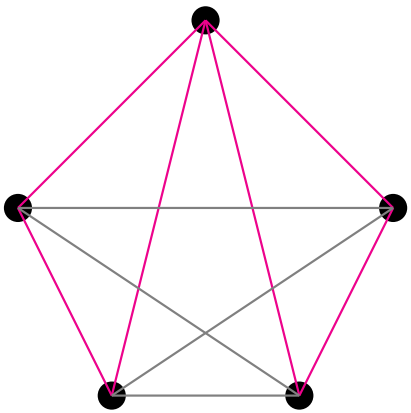

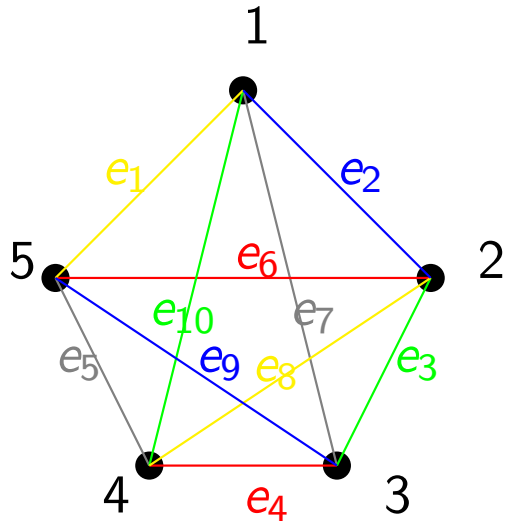

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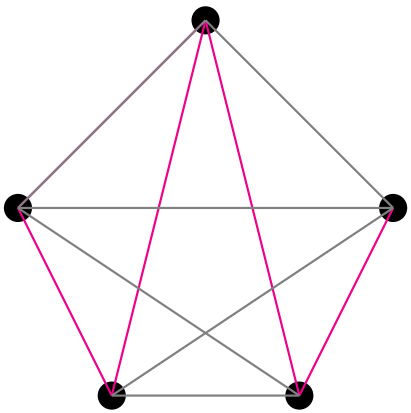

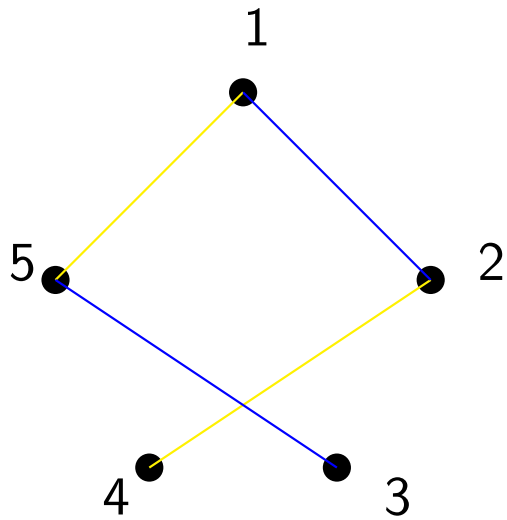

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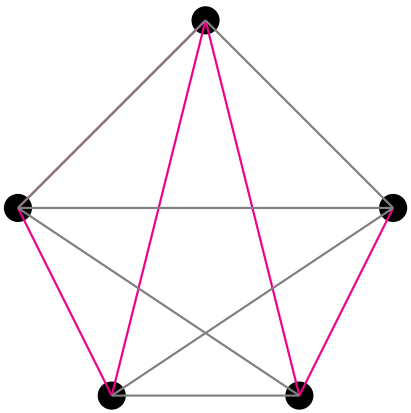

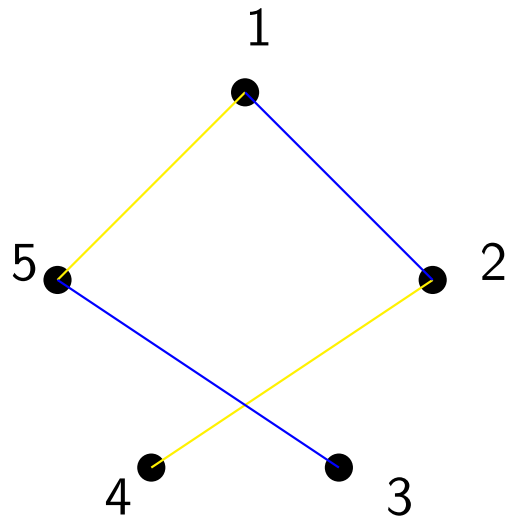

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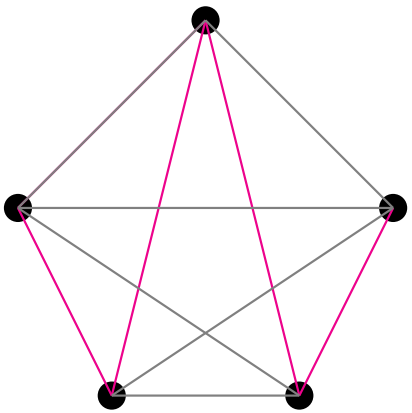

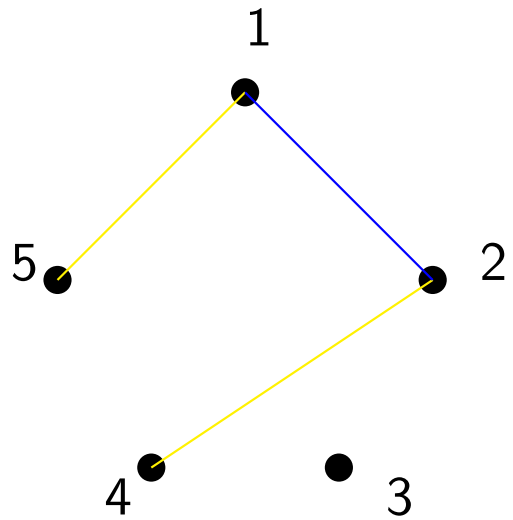

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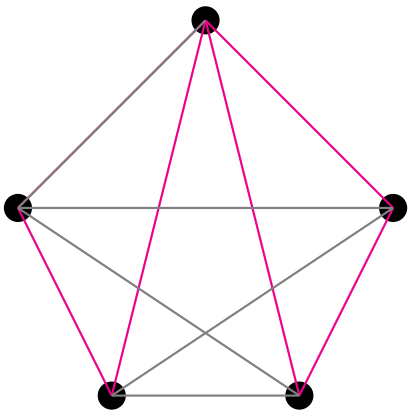

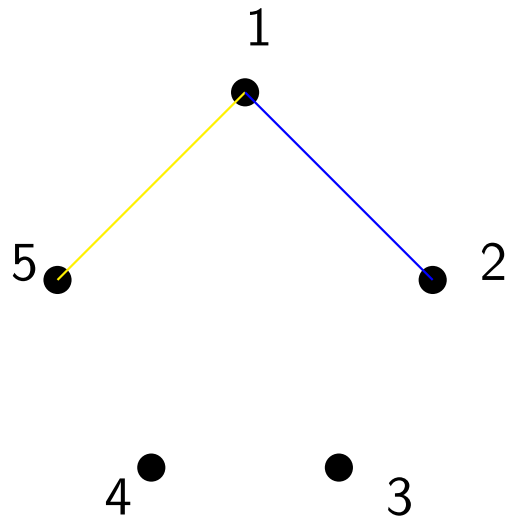

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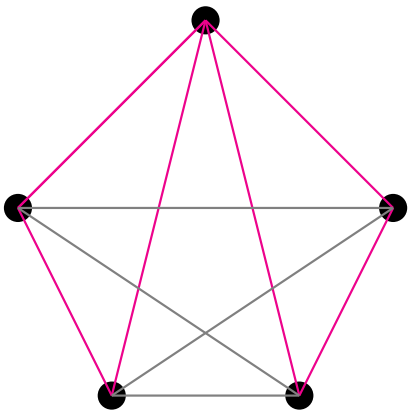

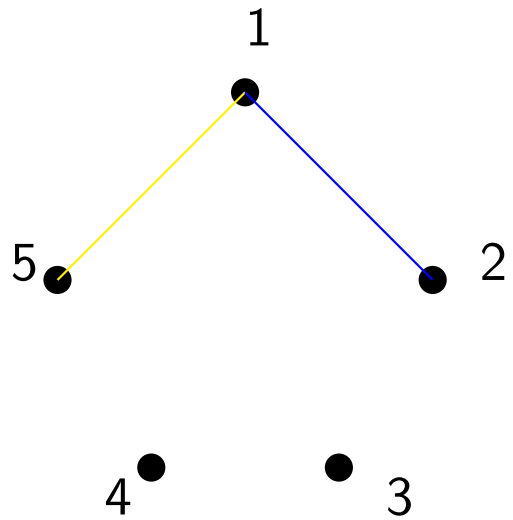

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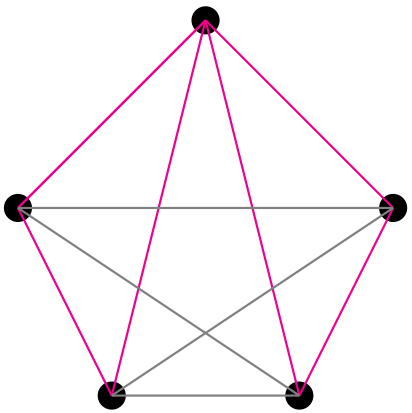
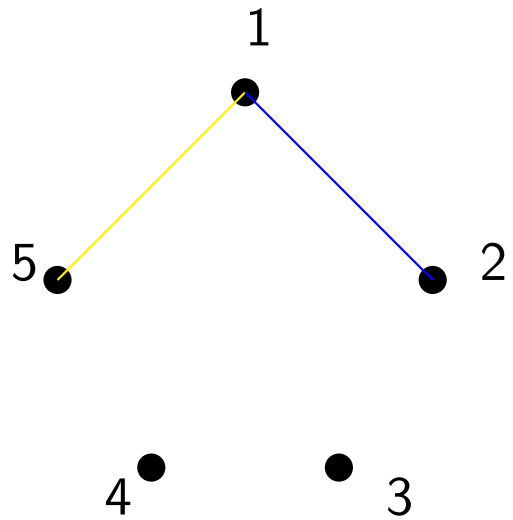
- The algorithm is linear in n.





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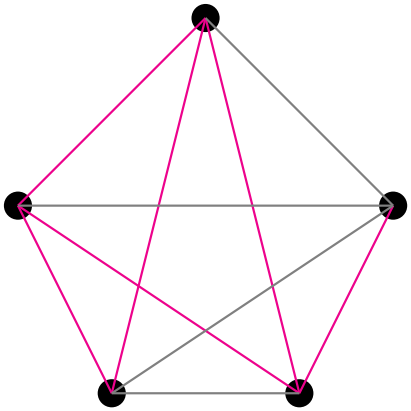

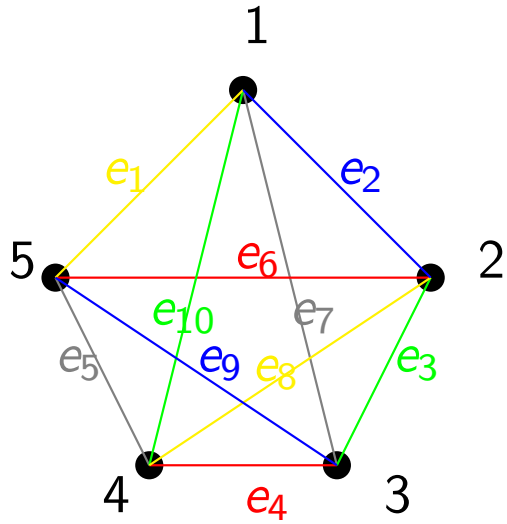

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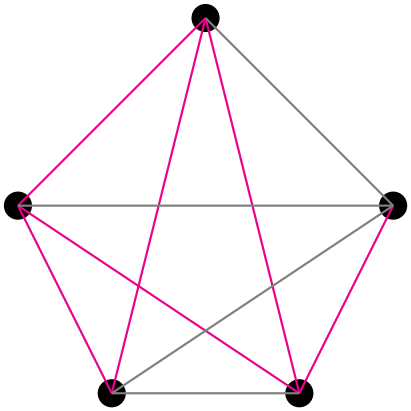

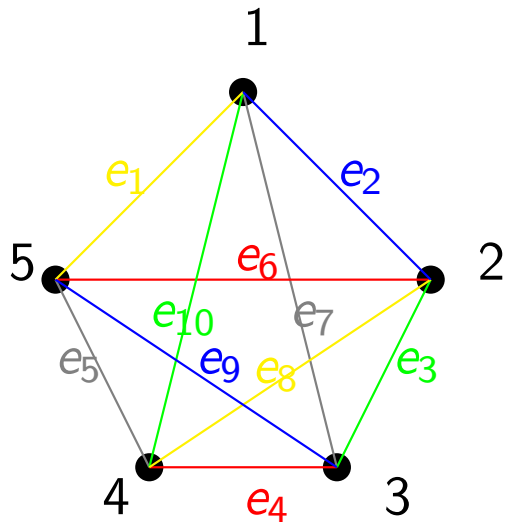

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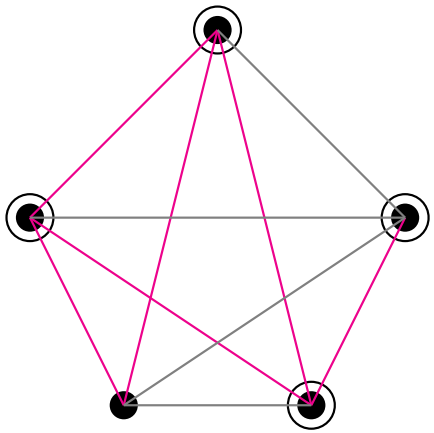

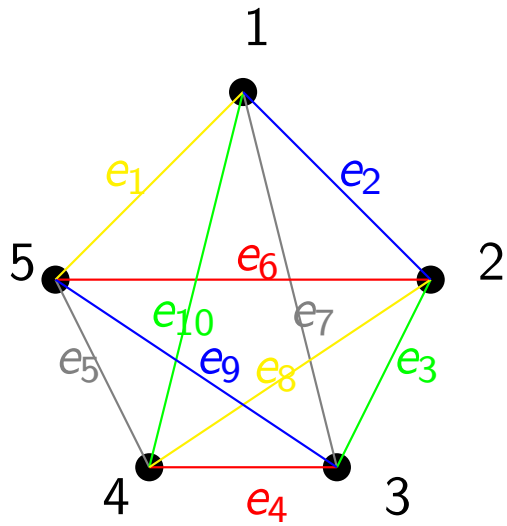

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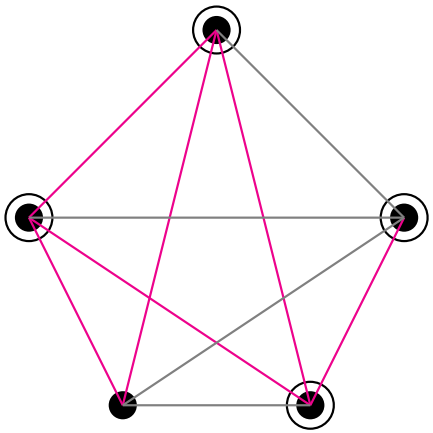

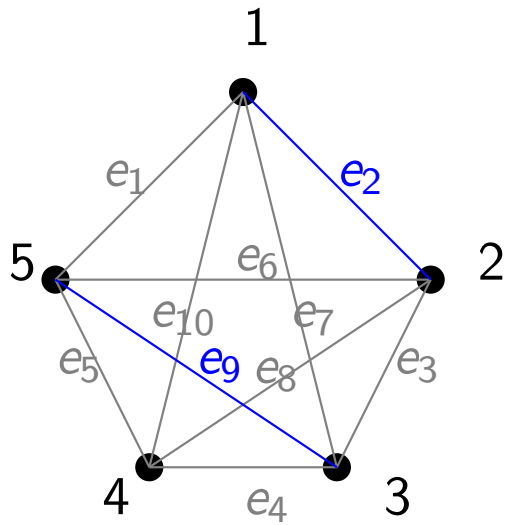

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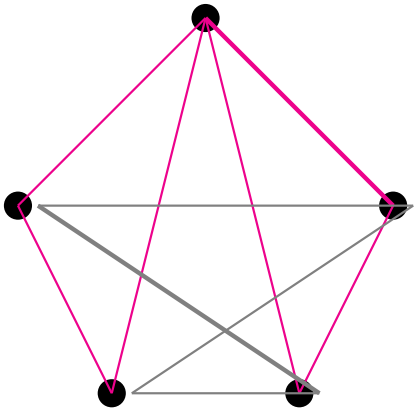

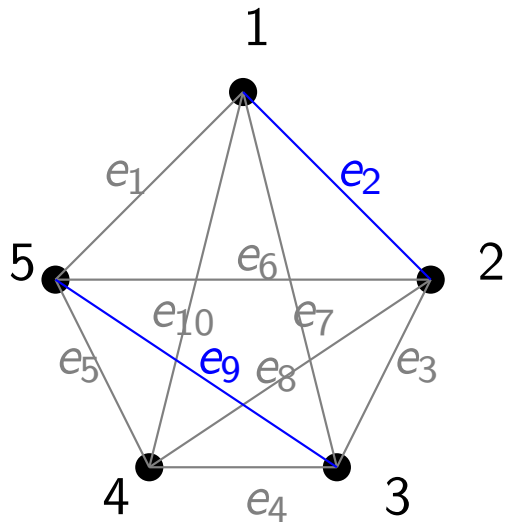

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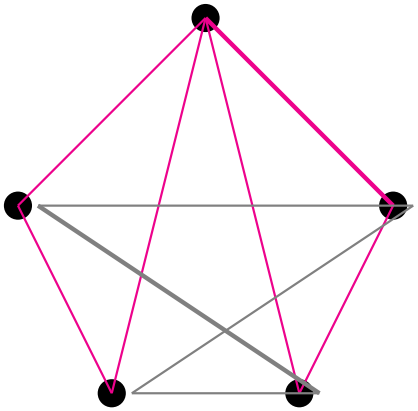

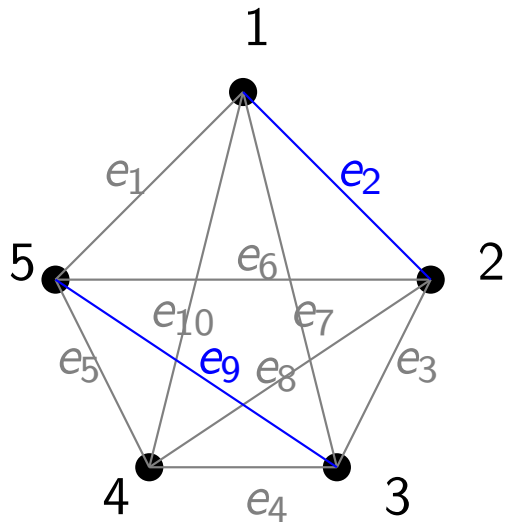

The dual B–codes

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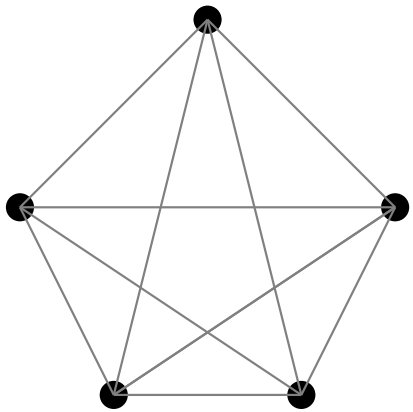

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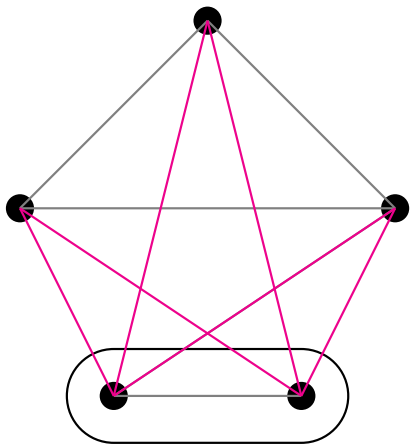

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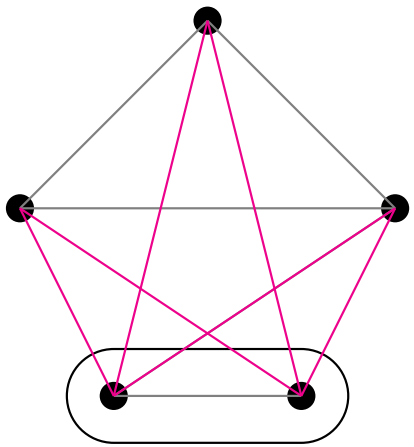

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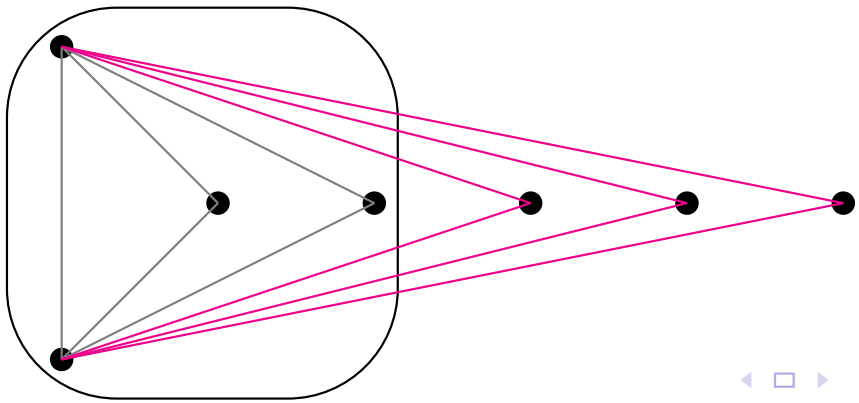

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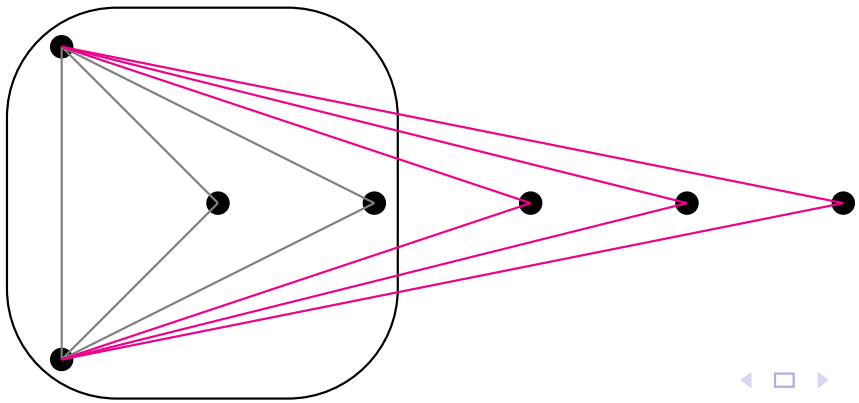

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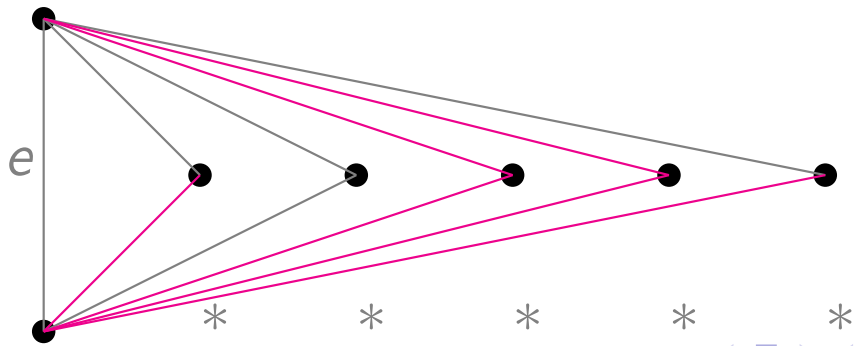

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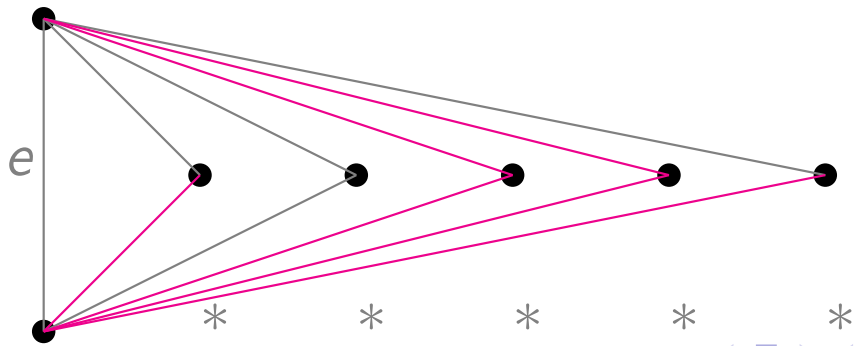

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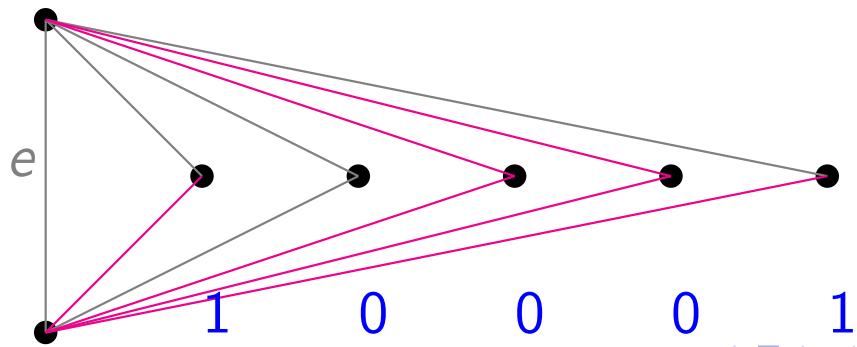

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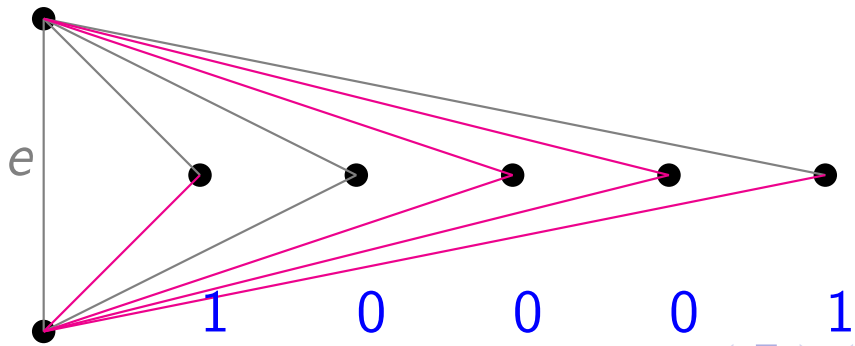
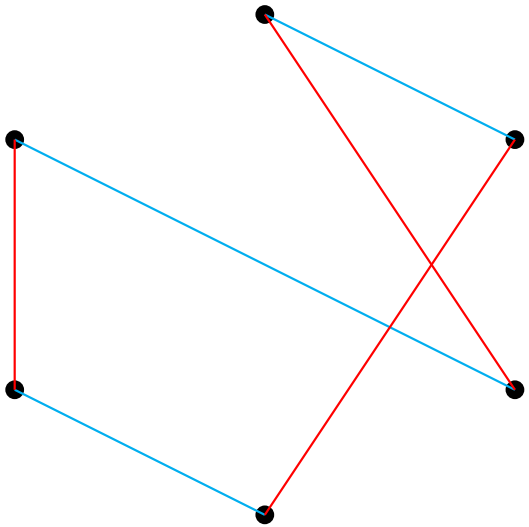
- Repeat for each edge incident to one endpoint of e.
- The algorithm is linear in $N = n(n-1)/2$ (the length of the code).

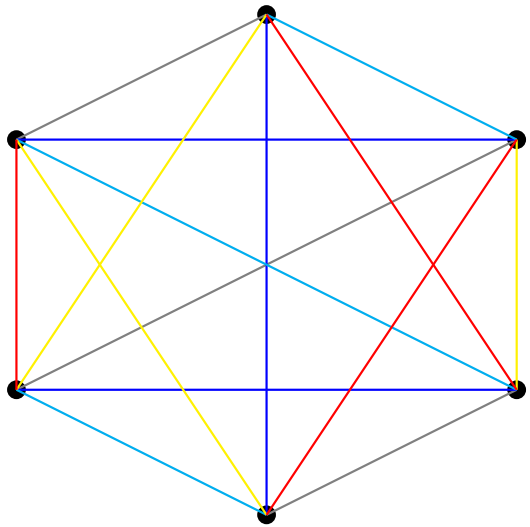


B–codes and the Perfect One-Factorization Conjecture

Conjecture (Kotzig, 1963)

For every n there is a proper coloring of the edges of K_{2n} with $2n-1$ colors such that every two colors induce a Hamiltonian cycle. Equivalently, there is a coloring of K_{2n-1} with $2n-1$ colors such that every two colors induce a Hamiltonian path.





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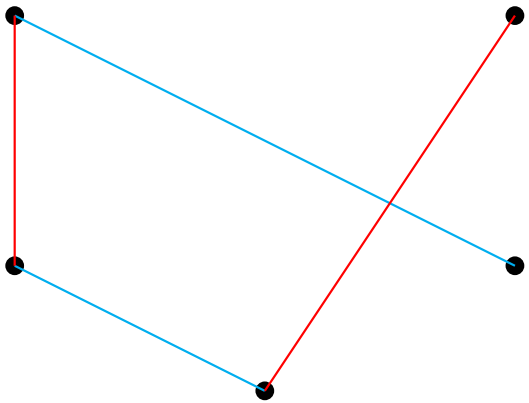

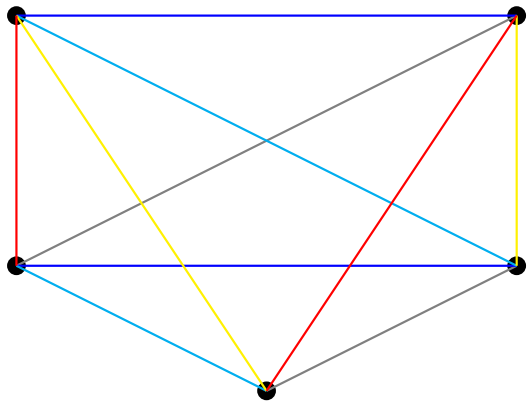

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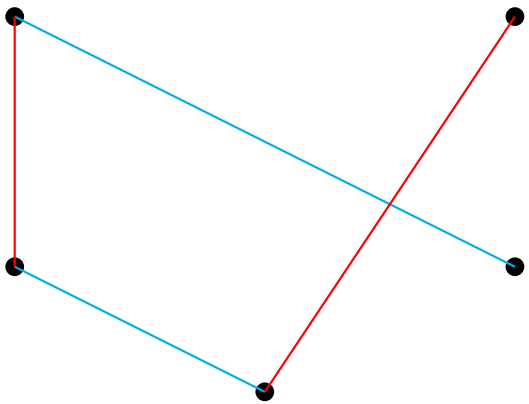

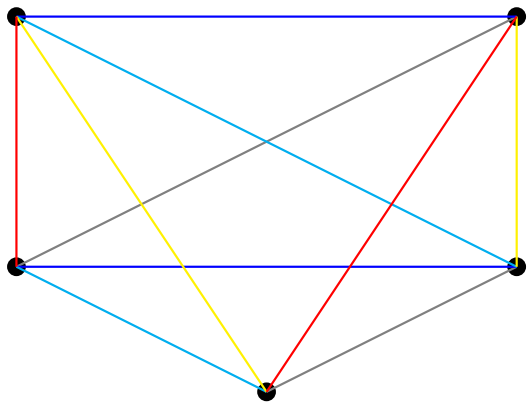

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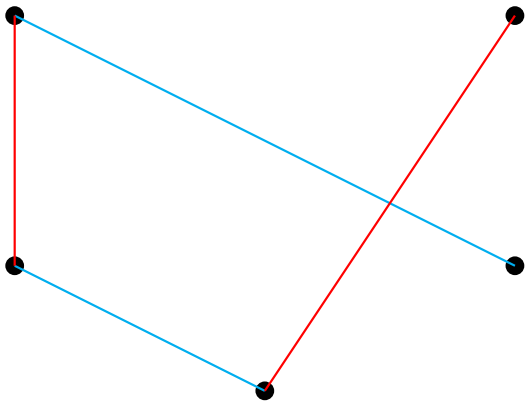

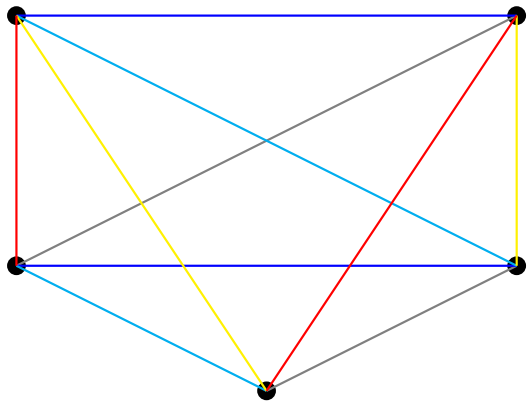

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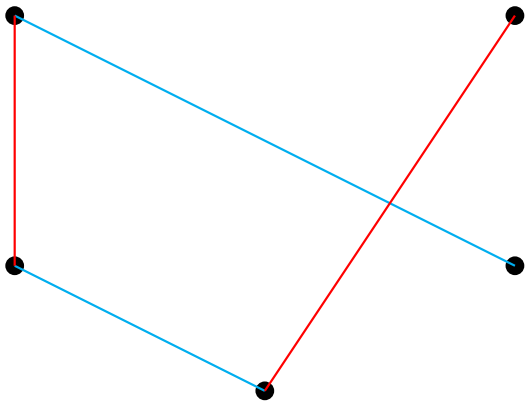

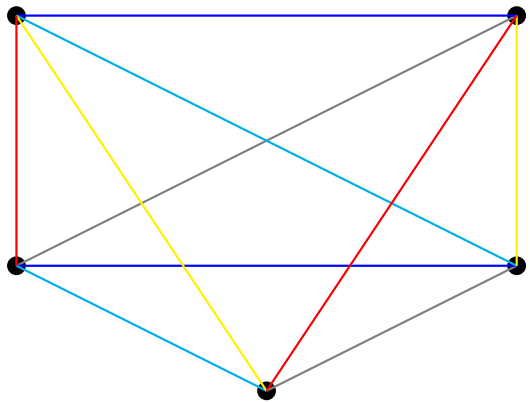

More examples of MDS array cycle codes

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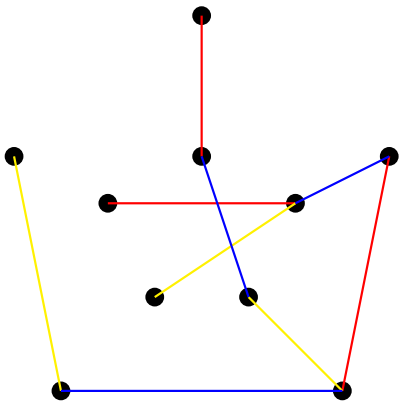

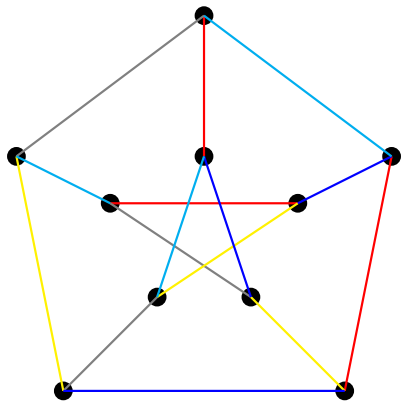

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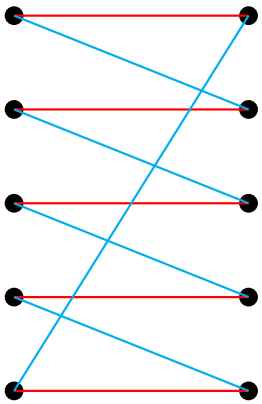

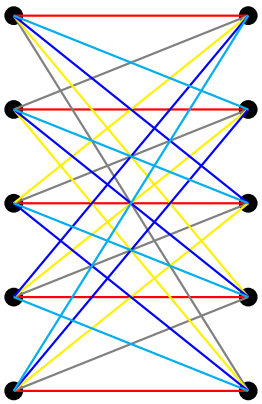

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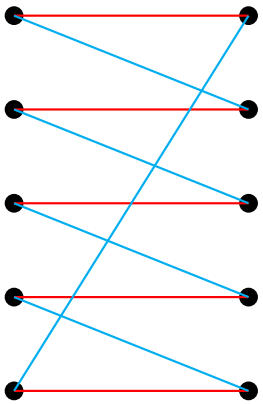

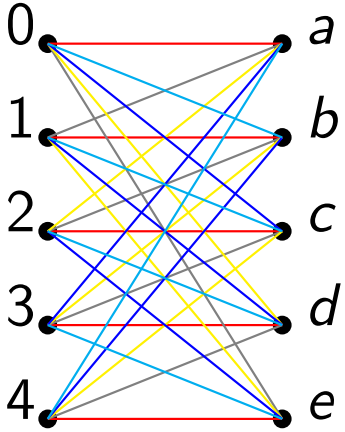

$$\begin{bmatrix} 0a & 2d & 4b & 1e & 3c \\ 3d & 0b & 2e & 4c & 1a \\ 1b & 3e & 0c & 2a & 4d \\ 4e & 1c & 3a & 0d & 2b \\ 2c & 4a & 1d & 3b & 0e \end{bmatrix}$$

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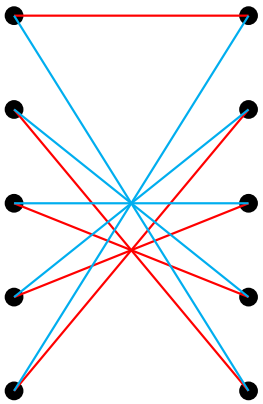

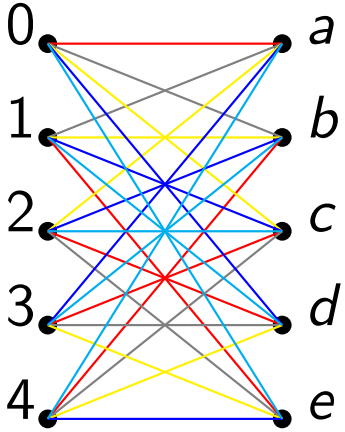

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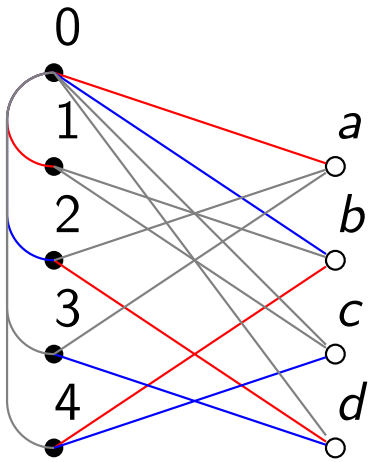
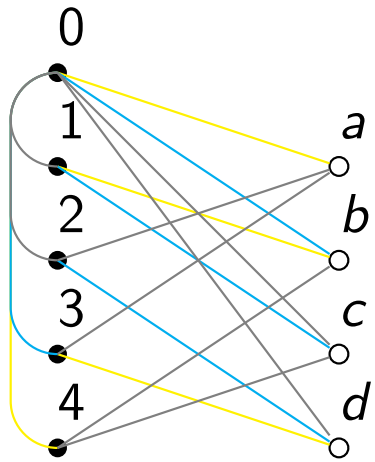
- For every prime p there is a graph G with $(p-1)^2$ edges and $2p-1$ vertices which admits a double array coloring.



$$\begin{pmatrix} 0a & 3d & 1b & 04 \\ 2d & 0b & 03 & 1c \\ 4b & 02 & 0c & 3a \\ 01 & 4c & 2b & 0d \end{pmatrix}$$



An application to Secret Sharing Schemes

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- If only $n-3$ participants meet, every arbitrary filling of the secret column allows the algorithm to recover a codeword.

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- Choose a codeword at random from an MDS Array Cycle Code on K_n.

$$\begin{pmatrix} 1 & 1 & * & * & 0 \\ 0 & 0 & * & * & 1 \end{pmatrix}$$

- Declare a column to be the secret and distribute the remaining columns to $n-1$ participants.
- If $n-2$ participants meet, the two–columns erasure algorithm allows them to recover (efficiently) the secret.
- If only $n-3$ participants meet, every arbitrary filling of the secret column allows the algorithm to recover a codeword.

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- If only $n-3$ participants meet, every arbitrary filling of the secret column allows the algorithm to recover a codeword.
  Every set of less than $n-2$ participants has 0–information of the secret: all values are equiprobable

Summary

- Array codes are designed for column erasures/errors (runs of errors).
- Graphs provide MDS array codes with minimum distance $D = 3$ (and their duals) with simple and efficient algorithms for erasure/error corrections.
- Array cycle codes graphs are available in sizes $((n-1)/2, n)$ and $(n-1, n-1)$ for $n = p, 2p-1, \ldots$ (connected to P1F Conjecture).
- Array double cycle codes of size $(p-1, p-1)$ are available and allow for row/column erasure/error correction.
- The codes provide an application to cryptographic schemes.

Th*nk you